AL52 - A New Mechanical Model of Magnetohydrodynamic Instabilities in Aluminium Electrolysis Cells

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Abstract

Understanding the magnetohydrodynamic metal pad instability (MPI) is important for aluminium electrolysis cells to operate stably and at a lower anode-to-cathode distance (ACD). Modelling the instability analytically requires solving a complex coupled system of fluids and electromagnetic equations. Pure analytical techniques are limited by the simplifying assumptions that make the equations solvable, but less representative of a real cell. Numerical simulations accounting for the great complexity of magnetohydrodynamics are much better at modelling a real cell, but often are expensive and can be time consuming. Another approach is using a simple mechanical model of the MPI, such as the compound pendulum given in [1] and expanded to include an oscillating current in [2]. This comparatively simple model offers great physical insight into the MPI and its mechanism but makes incorrect predictions about the parameter values that cause instability, when compared to real cells. In this work, we present a new mechanical model of the MPI in the form of a mobile with a variable center of mass. This new model shows instability at realistic cell parameters.

Keywords: Magnetohydrodynamic instability, Aluminium cell modelling

1. Introduction

Aluminium (Al) is produced using the Hall-Héroult process in Al electrolysis cells, where a molten cryolite layer floats atop a molten Al layer and both fluid layers are situated in between a carbon anode at the top and a carbon cathode at the bottom [3]. Alumina is dissolved in the cryolite, and a vertical electric current passes from the anode through the fluid layers to the cathode, decomposing the alumina and producing Al [4]. Due to the cryolite's very low electrical conductivity (~233 S/m), a lot of the electrical energy is transformed into heat by Joule heating [5] and produces no Al. Reducing the cryolite thickness, quantified by the anode to cathode distance (ACD), would be an effective way to reduce the energy loss, but reducing it beyond a critical threshold makes the Al cell unstable [3].

Natural disturbances exist on the cryolite-Al interface and can be thought of as a sum of interfacial wave modes that are decoupled [1]. When there is no current in the system and consequently there are no electromagnetic forces, the (purely gravitational) interfacial waves die out due to viscous damping. However, when current is present, electromagnetic forces are produced by the interaction between current and induced magnetic field. Horizontal electromagnetic forces may amplify waves present on the cryolite-Al interface [4], causing a circulating wave that can grow until the Al shorts to the anode [3]. The instability is magnetohydrodynamic (MHD) in nature, known as the metal pad instability (MPI), where the electromagnetic forces - couple interfacial gravitational wave modes having close frequencies leading to instability [1, 6]

A mechanical model of the MPI was given in [1] and then used again in [2]. The model is a compound pendulum, shown in Figure 1. It consists of a broad and thin Al plate connected by a rigid massless strut to a fixed ceiling. The Al plate can swing about the two horizontal axes (x and y axes), with the pivot at the connection point between the strut and the fixed ceiling. The gap between the Al plate and the fixed ceiling is filled with frictionless cryolite creating a path for current to pass from the "anode" ceiling to the Al plate.

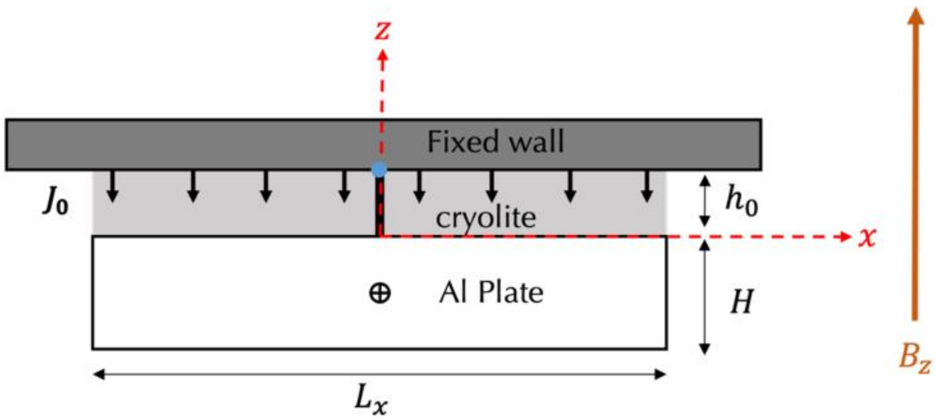

Figure 1. Pendulum model of the MPI used in [2] at equilibrium state. The Al plate's center of mass (COM) is half-way through its thickness and is marked by a circle with a cross. The pivot point is indicated by a blue disc.

where:

h_0	Thickness of the cryolite or ACD, m
H	Thickness of the Al plate (analogous to metal pad height), m
L_x	Length of the Al plate in the x direction, m
J_0	Nominal current density in bath, A/m^2
B_z	Vertical constant and uniform magnetic flux density, T.

If there is no current and we tilt the Al plate by a small amount (perturb it) from its equilibrium position (shown in Figure 2), gravity acts as a restoring force pushing the Al plate back towards the equilibrium position. This makes the Al plate oscillate about the equilibrium position either indefinitely when friction is neglected, or till the Al plate rests at the equilibrium position when friction is considered. The Al plate oscillates at its natural gravitational frequency in each horizontal direction, and those frequencies are [2]:

$$\omega_x^2 = \frac{12g\left(h_0+\frac{H}{2}\right)}{L_y^2} \tag{1}$$

$$\omega_y^2 = \frac{12g\left(h_0+\frac{H}{2}\right)}{L_x^2} \tag{2}$$

where:

ω_x	Natural gravitational frequencies of the Al plate in the x direction, rad/s
ω_y	Natural gravitational frequencies of the Al plate in the y direction, rad/s
L_y	Width of the Al plate in the y direction, m
g	Gravitational acceleration, 9.81 m/s^2.

Thus, when there is no current, the Al plate motion in the x and y directions is decoupled. However, when current is present, it interacts with the imposed uniform vertical magnetic flux density to generate horizontal electromagnetic forces which can couple the Al plate motion in the x and y directions. The electromagnetic forces modify the Al plate oscillation frequencies in the x and y directions, bringing them closer together. Strong enough electromagnetic forces can make these frequencies coincide, and the Al plate oscillates at the same frequency in both directions. If that happens, then the electromagnetic forces supply mechanical energy to the Al plate, making its motion unstable [1].

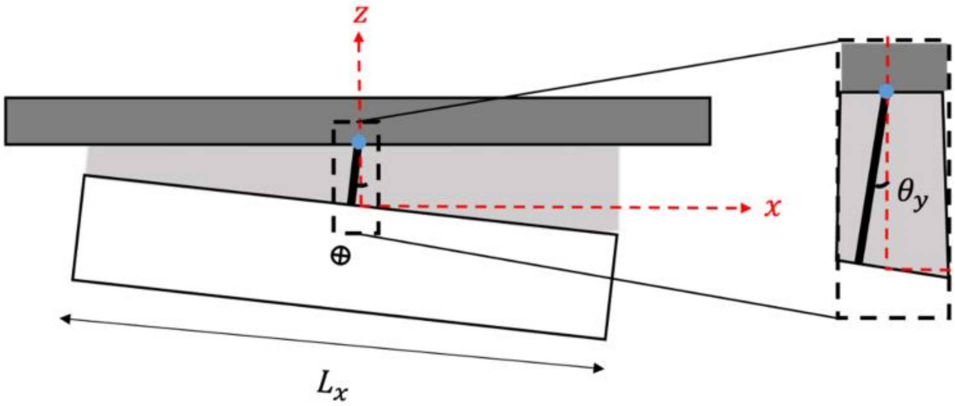

Figure 2. Pendulum model of the MPI at a small θ_y rotation, [rad], about y axis.

The strength of the coupling effect of the electromagnetic force is quantified by the coupling parameter [2]

$$a = \frac{(h_0 + H/2) J_0 B_z}{\rho_{al} H h_0} \tag{3}$$

where:

a Electromagnetic coupling parameter, 1/s^2

ρ_{al} Density of Al, kg/m^3.

When a reaches a_{crit}, (1/s^2), given by [2]

$$a_{crit} = \frac{|\omega_x^2 - \omega_y^2|}{2} = 6g\left(h_0 + \frac{H}{2}\right)\left|\frac{1}{L_x^2} - \frac{1}{L_y^2}\right| \tag{4}$$

instability occurs [1, 2]. A detailed derivation of Equation (3) and Equation (4) as well as an explanation of the instability mechanism can be found in [5].

The instability in the pendulum model is similar to the metal pad instability (MPI) in Al electrolysis cells, where electromagnetic forces couple interfacial gravity wave modes of similar frequencies [1]. Also, it offers great qualitative insight into the instability mechanism in Al electrolysis cells. For example, a higher current, stronger magnetic flux density, or lower ACD would lead to a larger electromagnetic coupling (a) which is a more unstable situation, as is the case in Al cells. However, this model fails to capture instability at realistic cell parameters. For example, consider a TRIMET 180 kA cell with the parameters shown in Table 1. Numerical magnetohydrodynamic simulations done using the MHD-Valdis software package in [3] showed that this cell is critically stable at 4.3 cm ACD and is unstable at 4.2 cm. Using the same parameters in the pendulum model and 2300 kg/m^3 for ρ_{al}, we find that a_{crit} is 0.47 s^{-2} and a is 0.18 s^{-2}, indicating that the model is very stable, as opposed to the actual cell. Here, we present a new mechanical model of the MPI that is unstable while considering realistic cell parameters.

Table 1. TRIMET 180 kA cell parameters, reproduced from [5].

J_0 (A/m²)	B_z (T)	h_0 (m)	L_x (m)	L_y(m)	H (m)
0.8	0.003	0.043	7.92	3.57	0.17

2. Mobile Model

2.1 Model Description

Consider a broad and thin rectangular Al plate of dimensions L_x, L_y, and H in the x, y and z directions, respectively. The Al plate is connected to a fixed ceiling by a massless, rigid strut, and can oscillate about the x and y axes with the pivot at the connection point between the Al plate and the strut, as shown in Figure 3. This is the first major difference from the pendulum model that had the pivot at the connection point between the strut and the ceiling – refer to Figure 1. This makes this model a "mobile" and not a "pendulum". The idea for using a mobile instead of a pendulum was first suggested by Marc Dupuis *et al.* in [7].

Another feature in the mobile model is that the COM of the Al plate is variable and is measured from the top of the plate as shown in Figure 3. For example, COM = 0.25 H means that the Al plate's center of mass is at a quarter of its thickness, measured from the top. This allows us to make the mobile model be more unstable by making the Al plate more top heavy, *i.e.*, by shifting the COM closer to the plate's top. We use the location of the COM as a tuning parameter in the model so that it is critically stable at the same ACD as a real cell.

The gap between the Al plate and the fixed ceiling has thickness h_0 and is filled with cryolite of negligible mass (representing the ACD in an Al cell). A steady and uniform current density of magnitude J_0 passes vertically downwards from the fixed ceiling through the cryolite to the Al plate where it is collected. A constant and uniform vertical magnetic flux density of magnitude B_z is imposed as shown by the orange arrow in Figure 3.

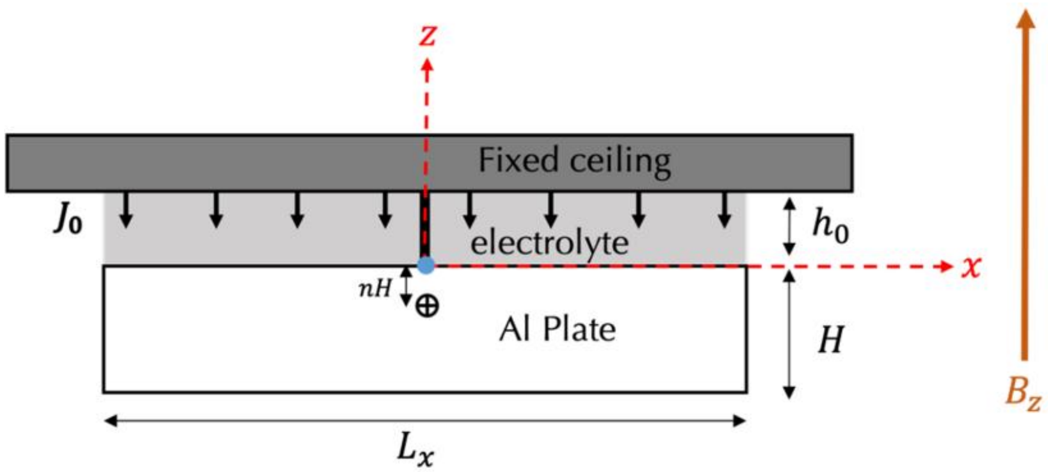

Figure 3. Mobile model of the MPI at equilibrium state. The pivot point is indicated by a blue disc and the Al plate's COM by a circle with a cross.

where:

n Relative position of the COM measured from the top, unitless.

2.2 Deriving Equations of Motion

We follow a similar procedure and assumptions to [2] and start by assuming a small rotation θ_x and θ_y (shown in Figure 4) about the x and y axes, respectively. By applying Newton's second law of rotation – about the horizontal axes parallel to the x and y directions and passing through the point of contact between the strut and the ceiling – and rearranging the terms, we find that the mobile's equations of motion are:

$$\ddot{\gamma}_x + \omega_x^2 \gamma_x = -a\gamma_y \tag{5}$$

$$\ddot{\gamma}_y + \omega_y^2 \gamma_y = \quad a\gamma_x \tag{6}$$

where:

$$\gamma_x = \frac{\theta_x}{L_x^2} \tag{7}$$

$$\gamma_y = \frac{\theta_y}{L_y^2} \tag{8}$$

The natural gravitational frequencies of the mobile are

$$\omega_x^2 = \frac{12ngH}{L_y^2} \tag{9}$$

$$\omega_y^2 = \frac{12ngH}{L_x^2} \tag{10}$$

The electromagnetic coupling parameter is

$$a = \frac{(h_0 + nH)J_0 B_z}{\rho_{al} H h_0} \tag{11}$$

Its critical value a_{crit} can be calculated by substituting Equation (9) and Equation (10) into Equation (4):

$$a_{crit} = 6ngH \left| \frac{1}{L_x^2} - \frac{1}{L_y^2} \right| \tag{12}$$

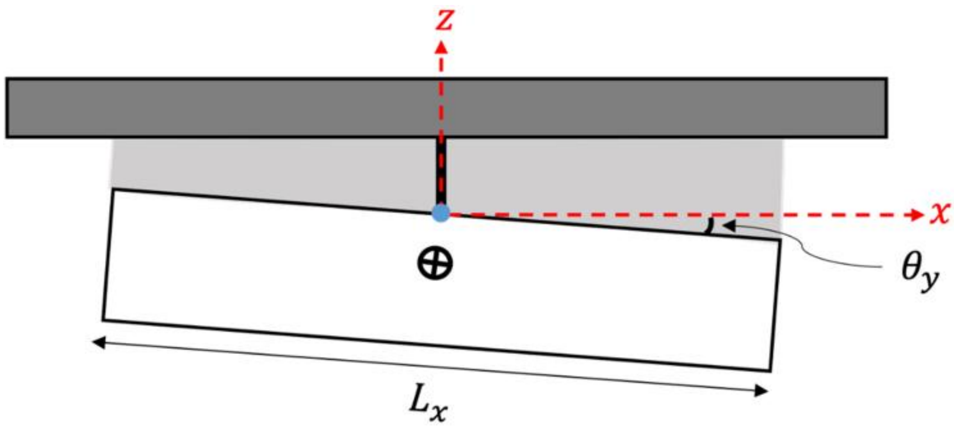

Figure 4. Mobile model of the MPI at a small θ_y rotation about y axis.

Notice that if we place the Al-plate COM at the center (n = 0.5) of the mobile, the electromagnetic coupling parameter (a) of the mobile model – Equation (11) – is the same as that of the pendulum model – Equation (3) – with the ACD (h_0) appearing in both. However, the ACD appears in a_{crit} for the pendulum model – Equation (4) – but not for the mobile model – Equation (12). This is an important simplification since for any positive ACD, a_{crit} of the mobile is always less than that of the pendulum model, while a is the same. This implies that the mobile model is always less stable than the pendulum model at a given ACD. Also, moving the COM of the Al plate closer to its top (decreasing n) has both a destabilizing effect because a_{crit} decreases linearly with n as shown by Equation (12), and a stabilizing effect because a also decreases with n as shown by Equation (11). We will see that this stabilizing effect is smaller than the destabilizing effect.

2.3 Finding COM Location for Critical Stability at Correct ACD

The stability of the mobile's motion is determined by the electromagnetic coupling parameter – Equation (11) – relative to its critical value –Equation (12). We want to adjust the COM location (n) of the Al plate such that the mobile's motion is critically stable (doesn't grow or decay in time) at 4.3 cm ACD. Using the same TRIMET cell parameters as before (Table 1), we calculated both a and a_{crit} at different Al plate COM locations (n) and the results are shown in Figure 5. For $n > 0.16$ (COM about a sixth of the Al plate thickness from the top), $a < a_{crit}$ and the mobile's motion is stable. While for $n \leq 0.16$, $a \leq a_{crit}$ and the mobile's motion is unstable. The boundary between the two regions at $n = 0.16$ is marked in Figure 5 by a red dashed line.

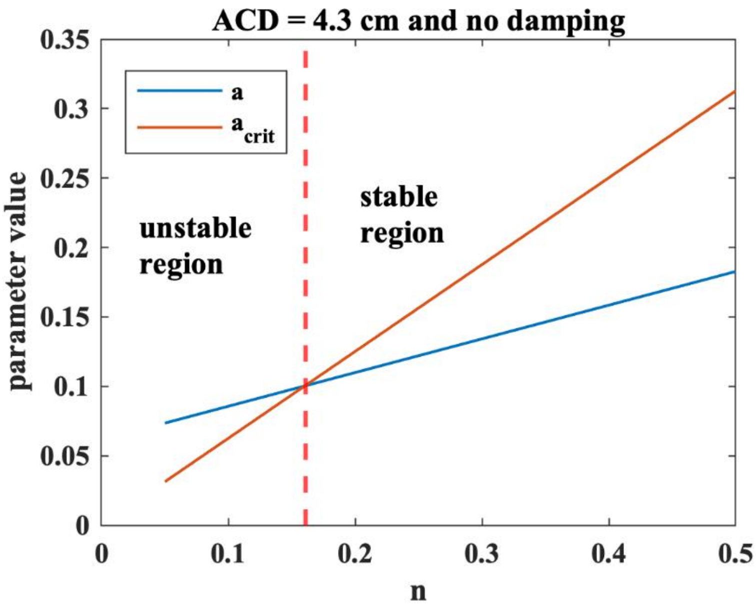

Figure 5. a and a_{crit} for the mobile model at different Al plate COM locations. The red dashed line indicates when $a = a_{crit}$.

We now set $n = 0.161$ and find the Al plate's motion by solving Equation (5) and Equation (6) numerically in the MATLAB software package. We use the initial condition for small angular tilt and no angular velocity, in both the x and y directions. The result is shown in Figure 6, where the mobile's oscillations do not grow or decay, as expected. The oscillations show a beating phenomenon [8] where the mobile oscillates at one frequency with its amplitude enveloped by a much lower frequency oscillation. This phenomenon physically occurs because the mobile is oscillating at two slightly different but distinct frequencies, which is expected for critical stability because as the electromagnetic coupling becomes stronger, the mobile's oscillation frequencies become closer to one another. Instability happens when the two frequencies coincide. The beating phenomenon is not representative of an actual Al cell which has damping effects such as those of viscosity.

Decreasing the ACD to 4.2 cm and solving for the mobile's motion, we see that the mobile is unstable now, with the oscillations growing exponentially in time as shown in Figure 6. Thus, we were able to create a mechanical model that can show instability at realistic cell parameters.

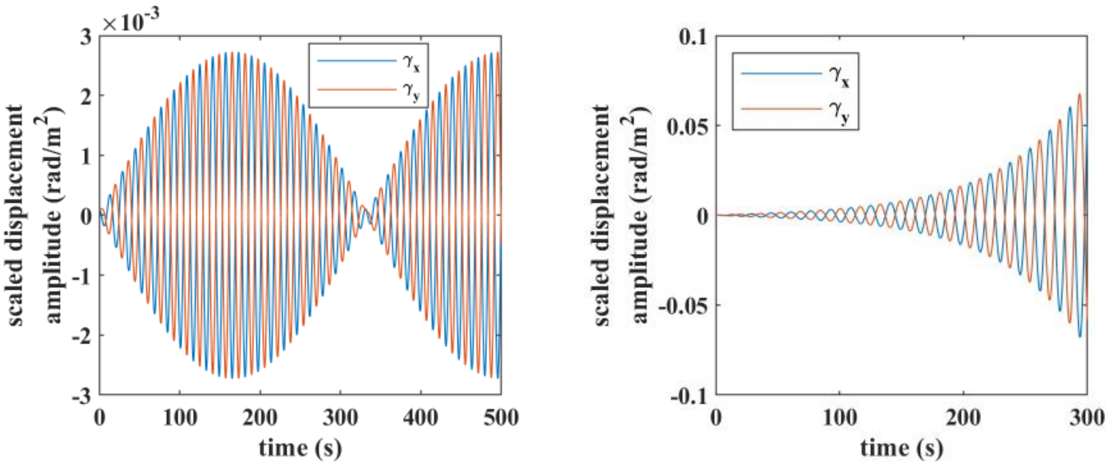

Figure 6. γ_x and γ_y for the mobile model with the Al plate's COM at n = 0.161. Left: 4.3 cm ACD, motion is critically stable. Right:4.2 cm ACD, motion is unstable.

3. Adding Friction to the Mobile Model

To make the mobile model even better at representing a cell, we added a frictional torque applied at the pivot to mimic the damping effects in an Al cell such as those of viscosity and the friction between the fluid layer and the electrode [1, 6]. The undamped mobile equations of motion are similar to that of a standard mass-spring vibrational system [9]. We chose to make the frictional torque proportional to the angular velocity of the Al plate and the natural gravitational frequency in each direction, as typical in a mass-spring-damper system [9]. Thus, the new equations of motion are

$$\ddot{\gamma}_x + 2\zeta\omega_x\dot{\gamma}_x + \omega_x^2\gamma_x = -a\gamma_y \tag{13}$$

$$\ddot{\gamma}_y + 2\zeta\omega_y\dot{\gamma}_y + \omega_y^2\gamma_y = \quad a\gamma_x \tag{14}$$

where:

ζ Damping coefficient, unitless

The Al cell interface wave model [10] showed that a pure gravitational oscillation (*i.e.*, when no current is present) is damped to 10 % of its amplitude within 1 period. We calibrate our damping coefficient (ζ) using this result, so we vary ζ to achieve 90 % damping of the pure gravitational oscillation within 1 period. The pure gravitational oscillations with no damping are shown in Figure 7, where the mobile oscillates at its natural gravitational frequency in each direction and the amplitude is constant. Adding a damping coefficient ζ of 0.35 reduces the amplitude to about 10 % of its initial value within 1 period as shown in Figure 7.

After finding the appropriate damping of ζ = 0.35, we solve for the mobile's motion by solving Equation (13) and Equation (14) numerically in MATLAB with the same initial conditions as before and 4.3 cm ACD. Due to the damping, the oscillations are now decaying in time as shown in Figure 8.

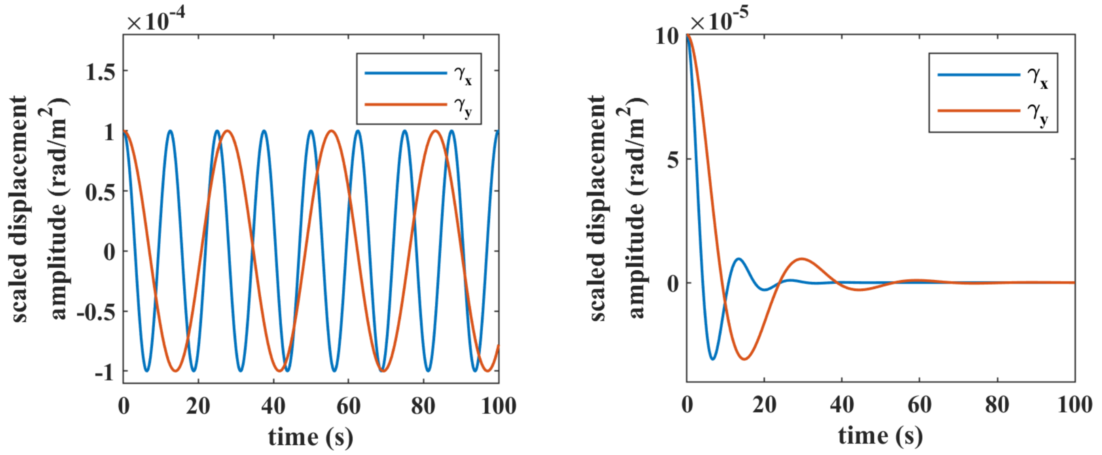

Figure 7. Mobile's motion in the x and y direction at 4.3 cm ACD and no current. Left: no damping is present ($\zeta = 0$), Right: damping of $\zeta = 0.35$ is applied and the oscillations decays about 90% in 1 period.

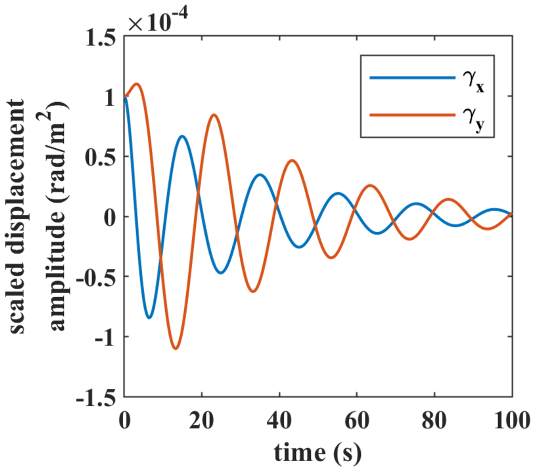

Figure 8. Mobile's motion in the x and y direction at 4.3 cm ACD with $\zeta = 0.35$. The oscillations decay in time.

To regain critical stability at 4.3 cm ACD, we move the Al plate COM closer to the top by decreasing n from 0.161 to 0.1182. As shown in Figure 9, after adjusting the plate's COM the oscillations at 4.3 cm do not grow or decay in time, indicating critical stability. Notice that the beating phenomenon is no longer present – compare to Figure (6) – due to coalescing of the oscillation frequencies with the added damping [8]. This is more representative of an actual Al cell which at critical stability does not show a beating but rather pure oscillations. At 4.2 cm ACD, however, the oscillations are growing in time, indicating instability.

We lower the ACD again to 4 cm and solve for the mobile's motion which is expected to be unstable. The results are shown in Figure (10). We fit an exponential function to the oscillation peaks of γ_y (shown in red) to find the growth rate of the motion which is 0.00325 1/s. A simulation of the TRIMET 180 kA cell with the same parameters at 4 cm ACD done in MHD-Valdis was reported in [3], where the growth rate of the interface root-mean-square displacement was found to be 0.002213 1/s. Although the growth rate from the mobile model is different than that from the simulation, the values are encouraging and perhaps with further improvement to the damping term calibration, can get closer. Also, the electromagnetic forces couple the mobile's motion in the x and y directions which is equivalent to coupling the (0,1) and (1,0) modes in an Al cell. This is the only coupling possible in the mobile model due to the rigid nature of the Al plate. However, the simulation results from [3] showed that the MPI in this cell is a coupling between the (2,0), (0,1) and (1,1) modes. There is also a difference in period where the mobile's period is ~ 21.5 s while that for the frequency in [3] for the MPI is ~38 s.

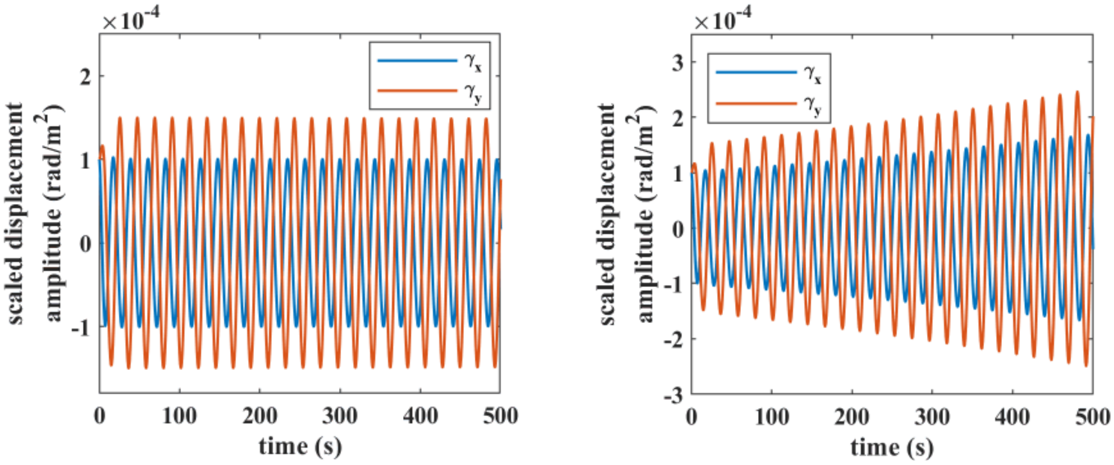

Figure 9. Mobile's motion in the x and y directions with $\zeta = 0.35$ and n = 0.1182. Left: at 4.3 cm ACD, Right: at 4.2 cm ACD.

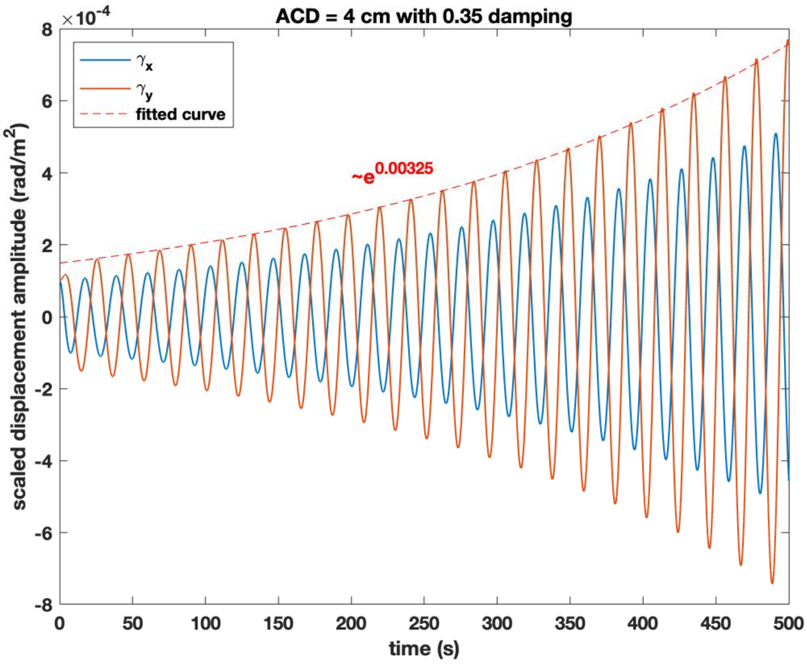

Figure 10. Mobile's motion in the x and y direction with $\zeta = 0.35$ and n = 0.1182 at 4 cm ACD.

4. Conclusion and Future Work

We presented a new mechanical model of the MPI: a mobile with variable center of mass and a damping term. The new model allows us to tune its center of mass so that critical stability can be achieved at the desired ACD. In practice, the desired ACD is found using the results of an ACD squeezing test. We tuned the mobile model to the parameters of a TRIMET 180 kA electrolysis cell used in [3] and showed that it exhibits critical stability at 4.3 cm ACD just like the actual cell. We also added a damping term and tuned it to achieve 90 % damping of the pure gravitational oscillation within 1 period [10]. By doing so, we were able to find the growth rate of the mobile's motion which was of the same order of magnitude as the one reported in [3] from a simulation of the TRIMET cell done in MHD-Valdis. Although not exact, the results are promising and could get closer with better calibration of the damping parameter ζ. The mobile model presents an opportunity to quickly explore the stability of the cell in a parameter space. More specifically, rather than running many computationally expensive numerical magnetohydrodynamic simulations to see if the Al cell is stable for multiple combinations of parameters, one can first check the stability of the mobile model of that cell by solving its equations of motion with the same combinations of parameters. This has the benefit of rapidly narrowing down the combinations of cell parameters to be simulated numerically, reducing the computational expense. The mobile model would be a complement tool to numerical magnetohydrodynamic simulations.

The mobile model is an improvement over the previous pendulum model since it shows stability/instability at realistic cell parameters. However, it is still missing important parameters and features that impact the magnetohydrodynamic stability of a realistic cell. For example, an actual Al cell is subjected to a vertical magnetic flux density generated by the adjacent pot rows, internal conductors, external conductors, and magnetization effects. Thus, the vertical magnetic flux density has a distribution in space – $B_z(x, y)$ – unlike the mobile model where it is assumed to be uniform. This could a potential avenue for improving the model.

Moving forward, we plan on tuning the model more so it can better capture the growth/decay rates of interfacial waves from an actual cell. Also, we plan on leveraging the mobile model to explore the phase space of the dynamic stabilization of the cell [3, 7]. In particular, we are interested in looking at the stabilization effect of adding an oscillating vertical magnetic flux density at different oscillation frequencies and amplitudes.

5. References

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